LOCALLY HOMOGENEOUS COMPLEX MANIFOLDS

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In this paper we discuss some geometric and analytic properties of a class of locally homogeneous complex manifolds. Our original motivation came from algebraic geometry where certain non-compact, homogeneous complex manifolds arose naturally from the period matrices of general algebraic varieties in a similar fashion to the appearance of the Siegel upper-half-space from the periods of algebraic curves. However, these manifolds are generally *not* Hermitian symmetric domains and, because of this, several interesting new phenomena turn up.

The following is a description of the manifolds we have in mind. Let $G_{\mathbf{C}}$ be a connected, complex semi-simple Lie group and $B \subset G_{\mathbf{C}}$ a parabolic subgroup. Then, as is well known, the coset space $X = G_{\mathbf{C}}/B$ is a compact, homogeneous algebraic manifold. If $G \subset G_{\mathbf{C}}$ is a connected real form of $G_{\mathbf{C}}$ such that $G \cap B = V$ is compact, then the G-orbit of the origin in X is a connected open domain $D \subset X$, and $D = G/V$ is therefore a *homogeneous complex manifold*. Let $\Gamma \subset G$ be a discrete subgroup such that the normalizer $N(\Gamma)$ intersects V only in the identity. Since Γ acts properly discontinuously without fixed points on D, the quotient space $Y = \Gamma \backslash D$ inherits the structure of a complex manifold. We shall refer to a manifold of this type as a *locally homogeneous complex manifold*.

One case is when $G = M$ is a maximal compact subgroup of $G_{\mathbf{C}}$. Then necessarily $\Gamma = \{e\}$, and $D = X$ is the whole compact algebraic manifold. These varieties have been the subject of considerable study, and their basic properties are well known. The opposite extreme occurs when G has no compact factors. These non-compact homogeneous domains D then include the Hermitian symmetric spaces, about which quite a bit is known, and also include important and interesting non-classical domains which have been discussed relatively little. It is these manifolds which are our main interest; however, since the

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