MEROMORPHIC FUNCTIONS WITH MAXIMAL DEFICIENCY SUM AND A CONJECTURE OF F. NEVANLINNA

BY

ALLEN WEITSMAN

Syracuse University, Syracuse, N. Y. and Purdue University, Lafayette, Ind.

I. Introduction

Let $f(z)$ be a meromorphic function and $\delta(\tau, f)$ be the deficiency, in the sense of Nevanlinna, of the value τ. The order λ and lower order μ of $f(z)$ are defined by the usual relations

$$\lambda = \limsup_{r\to\infty} \frac{\log T(r,f)}{\log r} \qquad \mu = \liminf_{r\to\infty} \frac{\log T(r,f)}{\log r},$$

and the total deficiency $\Delta(f)$ by

$$\Delta(f) = \sum_{\tau} \delta(\tau, f),$$

where the summation is to be extended to all values τ, finite or infinite, such that

$$\delta(\tau, f) > 0. \tag{1.1}$$

The number of deficient values, that is the number of distinct values of τ for which (1.1) holds, will be denoted by $\nu(f)$.

In addition to the familiar notations of Nevanlinna's theory, we shall find it convenient to define, for a measurable subset J of $[0, 2\pi)$ and a meromorphic function $g(z)$, the symbol

$$m(r, g; J) = \frac{1}{2\pi} \int_J \log^+ |g(re^{i\theta})|\, d\theta.$$

The present investigation centers around the classical second fundamental theorem of Nevanlinna's theory which asserts that the total deficiency of any meromorphic function $f(z)$ satisfies the inequality

$$\Delta(f) \leqslant 2.$$

The main contribution of this paper is the following

This research was supported in part by a grant from the National Science Foundation GP 7507.