

ON THE TORAL STRUCTURE OF LIE p -ALGEBRAS

BY

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Introduction

The purpose of this paper is to describe some of the structure of a Lie p -algebra in terms of its tori. General properties of tori are developed in section 2, where maximal tori and Cartan subalgebras are related, preservation of properties of tori under base field extension and under p -homomorphisms is studied and the invariance of the dimension of maximal tori in a solvable Lie p -algebra is proved. In section 3, exponentials are introduced for the purpose of studying the distribution of tori in a Lie p -algebra. The significance of this section is that it shows how a class of exponentials, sufficiently rich at characteristic 0 to express the conjugacy of Cartan subalgebras, can be effectively introduced at characteristic p . In section 4, it is shown that the maximal tori of a solvable Lie p -algebra over an algebraically closed field are conjugate.

The general prerequisites for the paper are contained in [4], [6]. It is well to mention here results of N. Jacobson [4], [5] and G. Seligman [6], [7] on tori, which are important for this paper, as well results of D. Barnes [1] and R. Block [2] on exponentials in Lie algebras $\mathcal{L}$ satisfying certain conditions on the degree of nilpotency of $\text{ad } \mathcal{L}^\infty$.

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1. Preliminaries

We are concerned with only finite dimensional Lie algebras and vector spaces over fields.

1.1 *Base field extension.* If $\mathcal{L}$ is a Lie algebra or vector space over F , then the extension

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