GROUPS RELATED TO COMPACT RIEMANN SURFACES

BY

CHIH-HAN SAH

University of Pennsylvania, Philadelphia, Penn., U.S.A. (1)

It has been known for a long time that the automorphism group of a compact Riemann surface of genus $g > 1$ is a finite group. Indeed, Hurwitz gave the upper bound of $84(g-1)$ for the order. In [22, 23], Macbeath described a procedure to construct Riemann surfaces attaining this upper bound. He observed that these surfaces arise from normal subgroups of finite index in a particular abstract group. This abstract group is a discontinuous subgroup of conformal automorphisms of the upper half plane having the signature $\{2, 3, 7 \mid 0\}$. By considering homomorphisms of this abstract group G into a finite group PSL $(2, p^f)$ Macbeath found many normal subgroups of finite index in G. Later, Lehner and Newman [19] used another method to obtain several of the groups and surfaces discovered by Macbeath. The purpose of the present paper is to apply the available techniques and results of finite group theory to the study of the finite quotient groups of discontinuous groups connected with Riemann surfaces.

Section 1 deals with the two-dimensional projective representations of a triangular group in an algebraically closed field K. (The assumption that K be algebraically closed will be seen to be unnecessary.) Indeed, there are only a finite number of inequivalent representations. However, for higher dimensional representations, this is no longer the case (see Corollary of Proposition 2.7). Theorem 1.5 shows that all discontinuous groups acting on the upper half plane with fundamental domains having finite areas behave almost like free groups. In Theorem 1.6, under very mild restrictions on the signature, we give explicitly the number of distinct torsion-free normal subgroups of a triangular group with certain prescribed factor groups. After the completion of this paper, A. M. Macbeath kindly informed us that our section 1 overlaps results of his in a paper that will appear shortly.

Throughout section 2 we use freely some rather complicated results of finite group theory. We assume a familiarity with standard arguments involving Sylow's Theorem and

(1) This research was partially supported by a grant from the National Science Foundation.