A new capacity for plurisubharmonic functions

by

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1. Introduction

If Ω is a complex manifold, let $P(\Omega)$ denote the plurisubharmonic functions on Ω. Each $u\in P(\Omega)$ is subharmonic with respect to every operator Δ_a which in local coordinates may be written in the form

$$\Delta_a=\sum a_{ij}\frac{\partial^2}{\partial z_i\partial \bar{z}_j}$$

where $a=(a_{ij})$ is a nonnegative Hermitian matrix. We wish here to exploit the fact that plurisubharmonic functions are simultaneously subharmonic with respect to several Laplacians to obtain some results on their local behavior which are stronger than those known for subharmonic functions. We are motivated by the equation

$$\left(\det\left[\frac{\partial^2 u}{\partial z_i\partial \bar{z}_j}\right]\right)^{1/n}=\frac{1}{n}\inf\{\Delta_a u\colon \det[a_{ij}]=1\}\tag{1.1}$$

for $u\in P(\Omega)\cap C^2(\Omega)$ (c.f. Gaveau [15]); this quantity, in some sense, estimates the extent to which u lies in the interior of $P(\Omega)$. Because of the geometric nature of the cone $P(\Omega)$, it seems that a "potential theory" which can describe the properties of $P(\Omega)$ must necessarily be nonlinear.

The operator $dd^c=2i\partial\bar{\partial}$ and its exterior powers $(dd^c)^j$ are invariant under holomorphic mappings. It is easily seen that in local coordinates

$$(dd^c u)^n=c_n\det\left[\frac{\partial^2 u}{\partial z_i\partial \bar{z}_j}\right]dV(z).\tag{1.2}$$

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