REGULARITY FOR A CLASS OF NON-LINEAR ELLIPTIC SYSTEMS

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In this article we demonstrate that the solutions of a certain class of non-linear elliptic systems are smooth in the interior of the domain. One example of this class of equations is the system

$$\operatorname{div}\,(\varrho(|\nabla s|^2)\nabla s_k) = 0 \quad 1 \leqslant k \leqslant m, \tag{0.1}$$

where ϱ is a smooth positive function satisfying the ellipticity condition $\varrho(Q)+2\varrho'(Q)q>0$, ∇ denotes the gradient, and $|\nabla s|^2=\sum_{k=1}^{m}|\nabla s_k|^2$. This type of system arises as the Euler–Lagrange equations for the stationary points of an energy integral which has an intrinsic definition on maps between two Riemannian manifolds; the equations are therefore of geometric interest. However, the method of proof also applies to the equations of non-linear Hodge theory, which have been studied by L. M. and R. B. Sibner [9]. These are systems of equations for a closed p-form ω, $d\omega=0$ and

$$\delta(\varrho(|\omega|^2)\omega) = 0, \tag{0.2}$$

where ϱ must satisfy the same ellipticity condition given earlier. The proof is presented in a form which covers both cases.

We shall prove regularity in the interior for solutions of systems which do not depend explicitly on either the independent variable or the functions, but only on the derivatives of the functions. An extension to a more general class of systems of the same type with smooth dependence on dependent and independent variables will be important for integrals which arise in Riemannian geometry and probably can be carried out without any radically different techniques. Homogeneous Dirichlet and Neumann boundary value problems may be treated by reflection; however, the regularity up to the boundary for

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