THE LOCAL STRUCTURE OF TWISTED COVARIANCE ALGEBRAS

BY

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1. Introduction

The fundamental problem in investigating the unitary representation theory of a separable locally compact group G is to determine its space $G\hat{}$ of (equivalence classes of) irreducible representations. It is known that when G is not type I, $G\hat{}$, with the Mackey Borel structure, is not standard, or even countably separated. This is generally interpreted to mean that the irreducible representations of such a group are not classifiable, and so the problem becomes to find a substitute for $G\hat{}$, simple enough to afford some hope that it can be described completely, yet complicated enough to reflect a significant part of the representation theory of G. Two promising candidates have been proposed, both defined using the group C^*-algebra $C^*(G)$ (which has the same representation theory as G): the space Prim G of primitive ideals of $C^*(G)$, which was shown by Effros [19] to be a standard Borel space in the Borel structure generated by the hull-kernel topology; and the space G_{nor} of quasi-equivalence classes of normal representations (traceable factor representations) of $C^*(G)$, shown by Halpern [35] to be standard in the Mackey Borel structure. (The results of [19] and [35] are actually valid for arbitrary separable C^*-algebras, not just those arising from groups.)

In the case that G is type I, both of these spaces may be naturally identified with $G\hat{}$. Striking evidence that they are natural objects of study may be found in the beautiful result [49] of Pukanszky, that for connected G they are "the same" in the sense that the map which associates to any element of G_{nor} the kernel of its members is a bijection of G_{nor} onto Prim $C^*(G)$. (It is easily shown that this bijection is in fact a Borel isomorphism.)

[1] Supported in part by an NSF Graduate Fellowship and by a Grant-in-Aid from the Graduate Division of the University of California, Berkeley. This paper formed a portion of the author's doctoral thesis, submitted to the Univ. of Calif. in September, 1976.