

# IMMERSION AND EMBEDDING OF PROJECTIVE VARIETIES

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## § 0. Introduction

Let  $f: X \rightarrow Y$  be a morphism of algebraic varieties defined over an algebraically closed field. If  $X$  and  $Y$  are nonsingular, and the induced map  $df: T(X) \rightarrow T(Y)$  of tangent bundles is a monomorphism, then  $f$  is called an *immersion*. A one-to-one immersion is

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<sup>(1)</sup> This paper is a revision of the author's thesis at Brown University, June 1976.