The Riemann-Roch theorem for complex spaces

by

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The subject of the present paper is the proof of the Riemann-Roch theorem for (possibly singular) complex spaces:

THE RIEMANN-ROCH THEOREM. *Denote by $K_0^{\mathrm{hol}}(M)$ the Grothendieck group of the category of all coherent sheaves on the complex space M and by $K_0^{\mathrm{top}}(M)$ the usual homology K-functor of the underlying topological space. Then there exists a group homomorphism $\alpha_M: K_0^{\mathrm{hol}}(M) \to K_0^{\mathrm{top}}(M)$, such that:*

(a) For M regular the restriction of the homomorphism α_M to the subgroup $K^0_{\mathrm{hol}}(M) \subset K_0^{\mathrm{hol}}(M)$, generated by the classes of all locally free sheaves, coincides with the natural morphism

$$K^0_{\mathrm{hol}}(M) \to K^0_{\mathrm{top}}(M) \approx K_0^{\mathrm{top}}(M)$$

attaching to each locally free sheaf on M the class of the corresponding vector bundle.

(b) If $f: M \to N$ is a proper morphism of complex spaces, and $f_!: K_0^{\mathrm{hol}}(M) \to K_0^{\mathrm{hol}}(N)$ is the direct image homomorphism, provided by Grauert's theorem, then the equality

$$f_* \alpha_M(\mathscr{L}) = \alpha_N(f_! \mathscr{L})$$

holds for any coherent sheaf $\mathscr{L}$ on M.

A detailed consideration of this form of the Riemann-Roch theorem, and its relation to the classical form of this theorem, due to Hirzebruch and Grothendieck, can be found in [4].

Originally, the R-R theorem was proven by F. Hirzebruch for algebraic vector

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