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A STUDY IN THE UNIQUENESS OF HARMONIC FUNCTIONS.

BY

FRANTIŠEK WOLF

in STOCKHOLM.[1]

In this paper I want to deduce some uniqueness theorems for harmonic functions with assigned boundary values in the unit circle.

In this direction there exists a classical result for harmonic functions continuous on the boundary, based on the fact that harmonic functions take their extreme values on the frontier. Here, there was understood by a boundary value what we are going to denote by $u^D(z)$ (cf. 2.1). It was shown that a function, harmonic in a domain and such that $u^D = 0$ at all boundary points, vanishes identically. Even discontinuous boundary values, defined as limits along the radius, have been considered, especially by G. C. Evans, in his book on the logarithmic potential. The harmonic functions had to be restricted by one of the following majorants:

$$|u(r,\theta)| \leq M, \qquad \int_0^{2\pi} |u(r,\theta)|^p \, d\theta < M.$$

The aim of this paper is to consider (i) more general boundary values, such as u_D, defined in 2.2 or limits u_L along the radius, or even more general curves, defined in 6.0; (ii) more general majorants.

Thus we prove in 7.4.6 a result which in a simplified form runs;

If (i) $u(r,\theta)$ is harmonic in the unit circle,

(ii) at every boundary point θ_0, $u(r,\theta)$ converges to zero, if $(r,\theta) \to (1,\theta_0)$ in any sector (cf. def. in 1.0),

(iii) for every $\varepsilon > 0$, there is an $R < 1$ such that $|u(r,\theta)| \leq e^{\varepsilon/(1-r)^m}$ for $r > R$,

then $u \equiv 0$.

[1] Now: Macalester College, St. Paul, Minn., U. S. A.