UNITARY REPRESENTATIONS DEFINED BY BOUNDARY CONDITIONS—THE CASE OF $\mathfrak{sl}(2, R)$

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§ 1. Introduction

Let $\mathfrak{g}$ be a Lie algebra over R, the field of real numbers, and σ, a $\mathfrak{g}$-module in a Hilbert space $\mathcal{H}$. If the domain of σ is dense, one can define an adjoint module $\sigma^\dagger$ in $\mathcal{H}$ such that

$$(\sigma(a)f, g) = (f, \sigma^\dagger(a^\dagger)g)$$

for all $f \in \mathcal{D}(\sigma)$, $g \in \mathcal{D}(\sigma^\dagger)$, $a \in \mathcal{U}[\mathfrak{g}]$, (see Appendix A for notation and details). The module σ is said to be symmetric or (infinitesimally) unitary if $\sigma \subset \sigma^\dagger$ and self-adjoint if $\sigma = \sigma^\dagger$. The importance of self-adjointness comes from the fact that dT is a self-adjoint module (see Appendix A). Here T is a unitary representation of the simply connected group corresponding to $\mathfrak{g}$, and dT is the usual $\mathfrak{g}$-module with the set of C^∞-vectors of T as its domain. Calling a $\mathfrak{g}$-module exact if it is equal to dT for some T, a natural problem would be to determine all exact extensions of a given symmetric $\mathfrak{g}$-module. The theory here is analogous to the theory of self-adjoint extensions of a single unbounded symmetric operator. In fact if $\dim \mathfrak{g} = 1$, it is well known that $\mathfrak{g}$-module is exact if and only if it is self-adjoint. For the general case, self-adjointness is necessary but not sufficient for exact-

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