

THE POWEROID, AN EXTENSION OF THE MATHEMATICAL NOTION OF POWER.

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1. The essential property of the factorials occurring in finite differences is that, when the proper difference operator is applied to the factorial, we obtain the factorial of the preceding degree, while every factorial of degree > 0 vanishes for $x = 0$. It is this property which makes factorials suited to expansion purposes. Therefore the question naturally presents itself, to find the most general class of polynomials possessing this property. A still more general class has been considered by Aitken¹ who introduces a new operator at each step; but it appears that there is considerable advantage in investigating separately the case where all the operators are identical, because in that case we may avail ourselves of certain theorems belonging to the calculus of operations and thus obtain convenient explicit expressions for the polynomials.

It will be assumed in this paper that the reader is familiar with certain elementary notions belonging to the calculus of operations, such as the definition and general properties of omega-symbols and their sub-class the theta-symbols. For details, and for the notation which will be employed in this paper, the reader is referred to the work quoted below.²

Where nothing else is said, the object of the operations will be a polynomial,

¹ A. C. AITKEN: On a generalization of formulae for polynomial interpolation. *Journal of the Institute of Actuaries*, Vol. LXI (1930), p. 107. See also *Proceedings of the Edinburgh Mathematical Society*, Series 2, Vol. I (1929), p. 199.

² J. F. STEFFENSEN: *Interpolation* (Baltimore 1927), § 2 and § 18, or the same articles in the Danish edition of 1925.