THE ARITHMETIC COHEN-MACAULAY CHARACTER OF SCHUBERT SCHEMES

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1. In this paper we prove the following theorem. (For the notation, see Section 2.)

THEOREM 1. *Let R be a Cohen–Macaulay ring. Then then homogenous coordinate ring of the Schubert scheme $\Omega(a_1 \dots a_d)$ is Cohen–Macaulay of relative dimension*

$$\left[\sum_{i=1}^{d} a_i - \tfrac{1}{2} d(d+1) + 1\right].$$

The theorem was also proved by M. Hochster. For an announcement of his results, see [5].

It was proved in a weaker local case (see Theorem 12 below) by J. A. Eagon and M. Hochster in "Cohen–Macaulay rings, invariant theory and the generic perfection of determinental loci" (to appear, cf. [3]).

The proof below owes many of its ideas to Eagon and Hochster and to G. Kempf. Frequent discussions with S. Kleiman and T. Svanes have also been helpful. I am especially grateful to Kleiman for his patient help preparing this material.

The proof goes as follows. We assume by induction that the homogeneous coordinate rings of small Schubert schemes are Cohen–Macaulay (the smallest being empty). Given a Schubert scheme we intersect it properly with a given hyperplane. The intersection then breaks up into the union of smaller Schubert schemes in the way described by a classical formula of M. Pieri. (We derive this formula from a result of W. V. D. Hodge. Hodge's result is the central part of the paper and we prove it following a method of J.-I. Igusa.) A result similar to lemmas of Eagon and Hochster shows that since the smaller Schubert schemes are Cohen–Macaulay, their union is also. As the equation of the hyperplane is not a zero-divisor in the homogeneous coordinate ring of the bigger Schubert scheme, this ring is then itself Cohen–Macaulay.