JORDAN ALGEBRAS OF TYPE I

BY

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1. Introduction

Jordan, von Neumann, and Wigner [5] have classified all finite dimensional Jordan algebras over the reals. The present paper is an attempt to do the same in the infinite dimensional case. The following restriction will be imposed: we assume the Jordan algebras are weakly closed Jordan algebras of self-adjoint operators with minimal projections acting on a Hilbert space, i.e. are irreducible JW-algebras of type I.(1) The result is then quite analogous to that in [5], except we do not get hold of the Jordan algebra $\mathfrak{M}_3^8$ of that paper, as should be expected from the work of Albert [1]. We first classify all irreducible JW-algebras of type I_n, $n \geq 3$ (Theorem 3.9). These algebras are roughly all self-adjoint operators on a Hilbert space over either the reals, the complexes, or the quaternions. Then all JW-factors of type I_n, $n \geq 3$, will be classified (Theorem 5.2). In addition to those in the irreducible case we find an additional JW-factor, namely one which is the C^*-homomorphic image of all self-adjoint operators on a Hilbert space. JW-factors of type I_2 are studied separately (Theorem 7.1). They are the spin factors, and except when the dimensions are small, are exactly those JW-factors which are not reversible. Global results of this type are obtained in section 6. Finally we show that the von Neumann algebra generated by a reversible JW-algebra of type I is itself of type I (Theorem 8.2).

A *J-algebra* is a real linear space $\mathfrak{A}$ of self-adjoint operators on a (complex) Hilbert space $\mathfrak{H}$ closed under the product $A \circ B = \frac{1}{2}(AB+BA)$. Then $\mathfrak{A}$ is closed under products of the form ABA and $ABC+CBA$, A, B, $C \in \mathfrak{A}$ (see [4]). A *JC-algebra* (resp. *JW-algebra*) is a uniformly (resp. weakly) closed J-algebra. A *JW-factor* is a JW-algebra with center the scalars (with respect to operator multiplication). A projection E in a J-algebra $\mathfrak{A}$ is *abelian* if $E\mathfrak{A}E$ is an abelian family of operators. By a *symmetry* we shall mean a self-adjoint unitary operator. Two projections E and F in a JW-algebra $\mathfrak{A}$ are said to be *equivalent* if there

(1) In a forthcoming paper all irreducible JW-algebras will be shown to be of type I.