ON THE BOUNDARY THEORY FOR MARKOV CHAINS. II

BY

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§ 11. Introduction

This paper may be regarded as a new and fairly self-contained one attached to §§ 2–4 of [I].(2) These sections are entitled "Terminology and notation", "The boundary" and "Fundamental theorems" respectively. The rest of [I] is either contained in a more general treatment (§ 5, § 9 and parts of § 6), or may be set aside as special cases under additional hypotheses (parts of § 6, § 7 and § 8). In particular the whole idea of "dual boundary" is dispensed with here, though this is not to say it should be abandoned forever. References to [I] beyond § 4 will be pinpointed.

In sum, the case of a finite number of passable atomic boundary points (briefly: "exits") will be settled here. Namely: all homogeneous Markov chains satisfying Assumptions A and B′ [I; p. 25 and p. 50] will be completely analyzed, with regard to the stochastic behavior of the sample functions as well as the analytical structure of transition probabilities, in fact both at the same time. To be exact, it will also be assumed that:

ASSUMPTION C_1. *All Φ-recurrent states are merged into one absorbing state.*

ASSUMPTION D. *All exits are distinguishable.*

It is important to note the difference between C_1 above and the erstwhile Assumption C [I; p. 47] which would require the absence of any $\prod$-recurrent state and is a serious restriction. On the contrary, conditions C_1 and D may be justly regarded as unessential for the boundary theory; see respectively the discussion at the end of § 15 here and on p. 38 of [I].

A culminating result of the theory has been that of "complete construction", origi-

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(2) References in roman capitals are listed at the end of the paper; references [1] to [14] are to be found at the end of [I].