A non-standard ideal of a radical Banach algebra of power series

by

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1. Introduction

We will be concerned here with certain radical Banach algebras of power series. Let $\mathbf{C}[[z]]$ be the algebra of formal power series over the complex field $\mathbf{C}$. We say that a sequence of positive reals $\{w(n)\}$ is a *radical algebra* weight provided the following hold:

$$w(0)=1 \text{ and } 0<w(n)\leq 1 \text{ for all } n\in\mathbf{Z}^{+}; \tag{1.1}$$

$$w(m+n)\leq w(m)\,w(n) \text{ for all } m, n\in\mathbf{Z}^{+}; \tag{1.2}$$

$$\lim_{n\to\infty} w(n)^{1/n}=0. \tag{1.3}$$

If these conditions hold, it is routine to check that

$$l^1(w(n))\equiv\left\{y=\sum_{n=0}^{\infty} y(n)\,z^n : \sum_{n=0}^{\infty} |y(n)|\,w(n)<\infty\right\}$$

is both a subalgebra of $\mathbf{C}[[z]]$ and a radical Banach algebra with identity adjoined. Conditions (1.1) and (1.2) make $l^1(w(n))$ into a Banach algebra; Condition (1.3) is needed to give it exactly one maximal ideal. The norm is defined in the natural way: $\|y\|=\Sigma_{n=0}^{\infty}|y(n)|\,w(n)$. We shall generally refer to $l^1(w(n))$ as a radical Banach algebra. The multiplication is given by the usual multiplication of formal power series. The reader is referred to [3], [4], and [8] for background material on such algebras. Besides $l^1(w(n))$, there are obvious closed ideals in $l^1(w(n))$:

$$M(n)\equiv\left\{\sum_{k=0}^{\infty} y(k)\,z^k\in l^1(w(n)) : y(0)=y(1)=\ldots=y(n-1)=0\right\} \tag{1.4}$$