BOUNDARY BEHAVIOUR OF MEROMORPHIC FUNCTIONS OF SEVERAL VARIABLES

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0. Introduction

The aim of this paper is to prove an n-dimensional generalization of a theorem of R. Nevanlinna. This theorem says that a meromorphic function of bounded characteristic in the unit disc $\{z: |z|<1\}$ has (finite) nontangential limits in almost every point of the circumference $\{z: |z|=1\}$. A function f is of bounded characteristic if

$$\int_0^{2\pi} \log^+ |f(re^{i\varphi})|\, d\varphi = O(1) \quad (0 \leqslant r < 1) \tag{1}$$

and

$$\sum_j (1-|b_j|) < \infty \tag{2}$$

where the b_j's are the poles of f (counted with multiplicity). $\log^+$ stands for the maximum of log and zero.

As a matter of fact, Nevanlinna showed that functions of bounded characteristic can be represented as the quotient of two bounded holomorphic functions: $f=g/h$. Now, Fatou's theorem tells us that g and h have nontangential limits a.e. on the circumference; hence the theorem of Nevanlinna. To be quite rigorous, it should be added that, by a theorem of F. and M. Riesz, the boundary values of h are a.e. different from zero.

In several variables Fatou's theorem generalizes straightforwardly, not only to functions in the unit ball, but to functions defined in domains with smooth boundary. The point is that this theorem holds for bounded harmonic functions too, and there is much less difference between harmonic functions in $\mathbf{C}$ and in $\mathbf{C}^n$ $(n>1)$, then between analytic functions in $\mathbf{C}$ and in $\mathbf{C}^n$.