INTERSECTION PROPERTIES OF BALLS IN COMPLEX BANACH SPACES WHOSE DUALS ARE L_1 SPACES

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Introduction

In the paper [10], L. Nachbin discovered and exploited the basic connection that exists between intersection properties of balls and extension properties of linear operators. This connection has been most strikingly revealed in the paper [8] by J. Lindenstrauss. For the aim of the present work, we want to exhibit the following result of that paper: We say with Lindenstrauss that a normed space A has the n, k *intersection property* if for every collection of n balls in A such that any k of them have a non void intersection, there is a point common to all the n balls. If A has the n, k intersection property for any $n \geqslant k$, then A has the *finite k intersection property*. It is then proved in [8, Theorem 6.1 and Theorem 5.5] that for a real Banach space A, the following three properties are equivalent.

(i) The dual A^* of A is isometric to an L_1 space.

(ii) The space A has the $4, 2$ intersection property.

(iii) For any 3-dimensional normed space Y and any 4-dimensional normed space $X \supset Y$ such that the unit ball of X is the convex hull of the unit ball in Y and a finite number of additional points, there exists for every linear opeator $T: Y \to A$ a norm preserving extension $\tilde{T}: X \to A$.

We remark that it is essential in this characterization that the space A is a *real* Banach space. Already the space $\mathbf{C}$ of all comples numbers shows that (ii) can not be valid in the complex case.

The starting point of the present work was the observation that it suffices in property (iii) to take just one space Y and just one space X, namely $X = l_1^4(\mathbf{R})$ and $Y = \{(x_j) \in l_1^4(\mathbf{R}) \colon \Sigma\, x_j = 0\}$. In fact, what we observed was that a normed space A has the $n, 2$ intersection property if and only if every linear operator T from the space

$$H^n(\mathbf{R}) = \left\{(x_j) \in l_1^n(\mathbf{R}) : \sum_{j=1}^{n} x_j = 0\right\}$$