ON CAPILLARY FREE SURFACES IN A GRAVITATIONAL FIELD

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We present here the second part of our study of the equation

$$\operatorname{div}\left(\frac{1}{W}\nabla u\right) \equiv \sum_{i=1}^{n} \frac{\partial}{\partial x_i}\left(\frac{1}{W}\frac{\partial u}{\partial x_i}\right) = n\mathcal{H}(\mathbf{x}; u), \qquad W = (1+|\nabla u|^2)^{\frac{1}{2}}, \tag{1}$$

for a scalar function $u(\mathbf{x})$ over an n-dimensional domain Ω with bounding surface Σ. For information on physical background and smoothness hypotheses we refer the reader to the Introduction in [7], where we study the case $\mathcal{H}$ independent of u. The interest for the present work, in which $\mathcal{H}$ is permitted to depend on u explicitly in certain ways, centers on the capillary equation

$$Nu \equiv \operatorname{div}\left(\frac{1}{W}\nabla u\right) = \varkappa u \tag{2}$$

where $\varkappa \neq 0$ is a constant, under a boundary condition

$$\mathbf{T}u \cdot \boldsymbol{\nu} \equiv \frac{1}{W}\boldsymbol{\nu}\cdot\nabla u = \cos\gamma \tag{3}$$

where $\boldsymbol{\nu}$ is unit exterior normal on Σ and γ is prescribed (see [7]). However, we shall discuss considerably more general situations to which our methods apply.

§ 1

We impose on $\mathcal{H}(\mathbf{x}; u)$ in (1) a single requirement:

A : For any $\delta > 0$ there exists $M_\delta < \infty$, such that at least one of the conditions

A_1: $\{\mathcal{H} \leqslant \delta^{-1} \Rightarrow u \leqslant M_\delta\}$

A_2: $\{\mathcal{H} \geqslant -\delta^{-1} \Rightarrow u \geqslant -M_\delta\}$

A_3: $\{\mathcal{H} \leqslant \delta^{-1} \Rightarrow u \geqslant -M_\delta\}$

A_4: $\{\mathcal{H} \geqslant -\delta^{-1} \Rightarrow u \leqslant M_\delta\}$

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