BERNOULLI MEASURE ALGEBRAS

BY

GAVIN BROWN and WILLIAM MORAN

Department of Pure Matematics, Liverpool, England

1. Introduction

We make a detailed study of certain L-subalgebras of the algebra $M(\mathbf{T})$ of bounded regular Borel measures on the circle. These algebras constitute perhaps the simplest case in which one can investigate the interplay between the convolution measure algebra structure of $M(\mathbf{T})$ and the arithmetic of the underlying group $\mathbf{T}$. Of course there is nothing new in considering $M(\mathbf{T})$, more generally $M(G)$ for any locally compact abelian group G, as a Banach algebra and as an L-space, but the study of the blend of these structures (i.e. the *convolution measure algebra* approach) has gained considerable impetus in the last few years. In particular J. L. Taylor has, in a brilliant sequence of papers [19], [20], [21], [22], [23], [24], [25], located the "good" subalgebras of $M(G)$ (crudely those with group maximal ideal spaces) in terms of the so called critical points of the maximal ideal space $\Delta(G)$ of $M(G)$. However, for non-discrete G, the residual structure of $\Delta(G)$ is largely unexplored and a cardinal objective of work of the present kind is to obtain more information in this area.

The work of Yu. A. Šreider in [17] leads to a description of the elements of $\Delta(G)$ as *generalized characters* (see § 2). In particular $M(G)$ can be exhibited as an inductive limit of certain single generator L-subalgebras, then, by duality $\Delta(G)$ appears as a projective limit of simpler maximal ideal spaces. Moreover Taylor shows in [19], that, given any convolution measure algebra N (e.g. a single generator L-subalgebra of $M(G)$), there exists a compact abelian jointly continuous semigroup $\Sigma(N)$, the *structure semigroup* of N, such that N is embedded as a weak $*$ dense L-subalgebra of the measure algebra $M(\Sigma(N))$ and the complex homomorphisms of N are induced by the continuous semicharacters of $\Sigma(N)$. So far these general tools have had little impact on the discussion of the fine arithmetical structure of $\Delta(G)$, indeed Taylor exclaims in [25] that "generalized characters are clearly impossible to understand," thus we believe that the time is appropriate for specific local studies.

Of course there is no difficulty in describing the maximal ideal spaces of the subalgebras