A CUTTING AND PASTING OF NONCOMPACT POLYGONS WITH APPLICATIONS TO FUCHSIAN GROUPS

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1. Introduction

In several classical textbooks on Algebraic Topology and Discontinuous Groups [cf. 7, 12] one finds the technique of cutting and pasting finite polygons to a canonical form. With Theorem 1 available we can use this classical technique to prove some important classical theorems in the theory of fuchsian groups as well as prove some interesting new ones. Thereby, we not only gain new insight into the theory of fuchsian groups, but also unify part of the classical theory.

Most of our applications will be to infinitely generated fuchsian groups. We do, however, prove some missed theorems about finitely generated groups along the way.

We should like to mention here that the techniques in this paper, especially Theorem 1, have many more applications to infinitely generated fuchsian groups whose detail is currently being worked out.

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2. Definitions

A fuchsian group Γ will be a group acting discontinuously on the unit disk Δ . A set of generators for Γ will be denoted by $\{A_1, A_2, ...\}$. In this case we write $\Gamma = \langle A_1, A_2 ... \rangle$. A fundamental domain for Γ is a domain $D \subseteq \overline{\Delta}$, the closure of Δ , such that (1) ∂D is accessible (accessible means that one can draw an arc from a point in D to any point in ∂D , where ∂D is the boundary of D); (2) $\partial D \cap \Delta$ can be written as the union of a countable (possibly infinite) number of Jordan arcs, which are identified in pairs by elements of Γ ;