SUBDOMINANT SOLUTIONS OF THE DIFFERENTIAL EQUATION

$y'' - \lambda^2(x-a_1)(x-a_2)\dots(x-a_m)\,y=0$

BY

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I. Introduction

1. The solutions f_k and f_{-k}

In order to state the main results of this paper, we shall start with the differential equation

$$d^2y/dz^2 - P(z, c_1, c_2, \dots, c_m)\,y = 0, \tag{1.1}$$

where $c_1, c_2, \dots, c_m$ are complex parameters and

$$P(z, c) = (z - c_1)(z - c_2)\dots(z - c_m). \tag{1.2}$$

Let

$$\left\{\prod_{j=1}^{m}(1 - c_j/z)\right\}^{\frac{1}{2}} = 1 + \sum_{h=1}^{\infty} b_h(c)\,z^{-h}, \tag{1.3}$$

where $b_h(c)$ are homogeneous polynomials of $c_1, \dots, c_m$ of degree h respectively, and let us put

$$\mathcal{A}_m(z, c) = \begin{cases} z^{\frac{1}{2}m}\left\{1 + \sum\limits_{h=1}^{\frac{1}{2}(m+1)} b_h(c)\,z^{-h}\right\} & (m = \text{odd}), \\ z^{\frac{1}{2}m}\left\{1 + \sum\limits_{h=1}^{\frac{1}{2}m} b_h(c)\,z^{-h}\right\} + \dfrac{b_{1+\frac{1}{2}m}(c)}{1+z} & (m = \text{even}), \end{cases} \tag{1.4}$$

where

$$z^r = \exp\{r[\log|z| + i\arg z]\} \tag{1.5}$$

for any constant r. Previously, P. F. Hsieh and Y. Sibuya [7] constructed a unique solution

$$y = \mathcal{Y}_m(z, c) \tag{1.6}$$

of the differential equation (1.1) such that

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