ON UNIQUENESS QUESTIONS IN THE THEORY OF VISCOUS FLOW

BY

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1. Introduction

In this paper it is shown, by means of counter examples, that for some spatial domains the motion of a viscous incompressible fluid is not uniquely determined by the traditional initial and boundary conditions (i.e., by the applied external forces and by the values of the fluid velocity at an initial instant of time, at the boundary of the spatial domain, and at spatial infinity). In a positive direction, we prove uniqueness for the initial boundary value problem in some classes of spatial domains, and uniqueness for this problem in other classes of domains under appropriate auxiliary conditions. Regarding the uniqueness questions to be considered here, it will be shown that the situation is much the same for the problems of steady flow as for those of nonstationary flow, and much the same for the linear Stokes equations as for the nonlinear Navier-Stokes equations.

In some respects our results are at variance with those given in previous works on the subject, and in some other respects our results may appear at first to be not new. Among the most important papers on theoretical hydrodynamics are some investigations of the existence and uniqueness theory for the boundary value problems of viscous flow within various classes of generalized solutions. Rather remarkably, the uniqueness proofs for these previously studied generalized solution classes do not make use of any properties of the spatial domain, and so the uniqueness theorems for them have in many cases been given for an arbitrary spatial domain. This is the case in the celebrated works [21] of Ladyzhenskaya, [31] of Prodi, and [33] of Serrin, and also in the present author's papers [14, 16, 17]. The uniqueness theorems of these papers are misleading, however, because the classes of generalized solutions to which they apply have been defined in such a way as to exclude from membership, in some domains, some classical and physically important solutions.

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