NORMAL FAMILIES AND THE NEVANLINNA THEORY

BY

DAVID DRASIN

Purdue University, Lafayette, Ind., U.S.A.

I. Introduction

1. Let $\mathfrak{F}$ be a family of nonconstant holomorphic functions defined in the disc $\Delta = \{|z| < 1\}$. $\mathfrak{F}$ is said to be *normal* if every sequence of functions in $\mathfrak{F}$ either contains a subuniformly convergent subsequence, or contains a subsequence which converges subuniformly to the constant ∞. A family $\mathfrak{F}$ of meromorphic functions is normal when every sequence of functions of $\mathfrak{F}$ has a subsequence which is subuniformly convergent with respect to the chordal metric.

P. Montel [15] first realized the scope and coherence of these families, and used them to give a particularly unified treatment of Picard's great theorems, and Schottky's and Landau's theorems. The fact that these results were so intimately related led A. Bloch to the hypothesis that precisely those properties which reduce a function meromorphic in $\mathbf{C}$ $(=\{|z| < \infty\})$ to a constant, make normal a family of functions meromorphic in Δ.

2. The Nevanlinna theory of meromorphic functions has proved an effective means of studying the value-distribution of a single meromorphic function in $\mathbf{C}$. In particular, a recent paper of W. K. Hayman [8] contains several striking results of this type.

In view of Bloch's observation, Hayman asks [10] whether his results have normal family analogues; the present paper establishes an affirmative answer in the important special case of *holomorphic* functions. Of greater interest, however, is that by using the standard arguments of the Nevanlinna theory we are able to present a unified exposition of the major value-distribution criteria for normal families of holomorphic functions (compare especially the proofs of Theorem 5 here and Theorem 8 of [8]). An extension of a theorem of Montel, valid for families of *meromorphic* functions, is also obtained with little additional effort.

The major problem faced is the handling of what will be referred to as *initial value terms*; that is, terms which depend on the values of the function or its derivatives at the