k-MERSIONS OF MANIFOLDS

BY

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1. Introduction and statement of results

This paper contains a generalization of the Smale–Hirsch classification of immersions, and the Phillips classification of submersions.

Let M^n be an n-dimensional C^∞ manifold and W^p a p-dimensional C^∞ manifold. A C^∞ mapping f: $M^n \to W^p$ is called a k-mersion if its rank is $\geqslant k$ everywhere. The set of k-mersions, endowed with the C^1 topology, is denoted $C^\infty(M^n, W^p; k)$. A k-regular homotopy between k-mersions f and g is a continuous map G: $I \to C^\infty(M^n, W^p; k)$ such that $G(0)=f$ and $G(1)=g$.

A k-bundle map, ψ: $TM^n \to TW^p$ between the tangent spaces of M^n and W^p is a continuous fiber preserving map such that the restriction of ψ to any fiber is a linear map of rank at least k. The space of k-bundle maps with the compact open topology is denoted $T(M^n, W^p; k)$.

An n-mersion is an immersion, and an n-regular homotopy is usually called a regular homotopy. In 1958 and 1959, Smale [8], [10], published papers classifying immersions of spheres in euclidean spaces. Smale proved that if $n<p$, the regular homotopy classes of immersions of S^n in R^p are in one to one corresponence with the homotopy classes of sections of S^n into the bundle associated with TS^n whose fiber is the Stiefel manifold $V_{p,n}$ of n frames in p-dimensional euclidean space. Smale obtained this classification by proving a stronger result, namely, that the map d: $C^\infty(S^n, R^p; n) \to T(S^n, R^p; n)$ defined by $d(f)=df$ is a weak homotopy equivalence if $n<p$.

In 1959, Hirsch [3] extended this result to the case of immersions, $C^\infty(M^n, W^p; n)$, of a C^∞ manifold in another, where $n<p$ and ∂W^p is empty. Poenaru's exposition of this result [8] was the basis of Phillips' thesis, published in 1965 as [7]. Say that a manifold

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