POTENTIAL THEORY OF RANDOM WALKS ON ABELIAN GROUPS

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1. Introduction

Let $\mathfrak{G}$ be a locally compact Abelian group and let μ denote a regular probability measure on $\mathfrak{G}$. If $\{\xi_n, n \geqslant 1\}$ is a sequence of independent $\mathfrak{G}$ valued random variables each having μ for their probability law, then the *random walk* with initial point S_0 is the Markov chain $S_n = S_0 + \xi_1 + \dots + \xi_n$. If $\mathfrak{G}_0$ is the minimal closed subgroup of $\mathfrak{G}$ generated by the support $S(\mu)$ of μ, then $P_0(S_n \in \mathfrak{G}_0 \text{ for all } n \geqslant 1) = 1$, where $P_x(\cdot)$ denotes conditional probability given $S_0 = x$. Henceforth we will assume that $\mathfrak{G}_0 = \mathfrak{G}$. This entails no real loss in generality and is essential for the proper formulation of our results. In addition, throughout the first 13 sections of the paper we always assume that $\mathfrak{G}$ is also noncompact. For a compact $\mathfrak{G}$ the corresponding results (where meaningful) are far easier to establish. We will discuss these in our final § 14.

Basic notation and concepts used throughout the paper are listed in § 2. The reader should refer to this section while reading the introduction as the need arises.

A random walk is said to be *recurrent* if for some compact neighborhood N of 0, $\sum_{n=1}^{\infty} P_0(S_n \in N) = \infty$. Otherwise the walk is called *transient*. It is a known fact (see Loynes [7]) that for a recurrent walk $\sum_{n=1}^{\infty} P_x(S_n \in N) = \infty$ for all x and open sets N, while for a transient walk $\sum_{n=1}^{\infty} P_x(S_n \in K) < \infty$ for all x and compact sets K. Moreover (Loynes [7]) in a recurrent walk, $P_x(V_N < \infty) \equiv 1$ for all open sets $N \neq \emptyset$. A random walk is *nonsingular* if for some $n \geqslant 1$, $\mu^{(n)}$ has a nonsingular component relative to the Haar measure on $\mathfrak{G}$. For a nonsingular walk the sets N in the above statements may be taken to be Borel sets of positive Haar measure.

Briefly, our main goals in this paper are five-fold. First, to establish the renewal theorem for transient random walks on $\mathfrak{G}$. This will be done in § 4 and will be the only place that transient walks are discussed. The remainder of the paper is devoted to recurrent

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