

THE STRUCTURE OF A FINITELY GENERATED KLEINIAN GROUP

BY

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In his classical work on automorphic functions Poincaré introduced the distinction between Fuchsian groups and the more general class of Kleinian groups. The Fuchsian case proved to be much more tractable, and soon grew to an impressive theory with applications in several branches of mathematics. In contrast, the study of Kleinian groups progressed slowly and still presents major unsolved problems. A recent surge of interest occurred when it was found that the theory of quasiconformal mappings has important bearing on the subject.

The writer proved (Ahlfors [1]), with minor restrictions that were subsequently removed by Greenberg [7] and Bers [5], that the orbit space of a finitely generated Kleinian group, when completed, is a finite union of compact Riemann surfaces. This result is the starting point of newer developments.

An important step was taken by Bers who, in several papers [3, 4, 5], emphasized the use of differentials of arbitrary order $q \geq 2$. This served to eliminate the deficiencies of the original approach, which, motivated by the connection with quasiconformal mappings, worked exclusively with quadratic differentials.

This paper continues the research of Bers, especially as developed in [4]. The reader may wish to consult Ahlfors [2] for notations, which differ from those of Bers, and for an assembly of elementary facts. The present paper is, however, essentially selfcontained.

The paper of Bers, at least indirectly, and my own, very directly, owe much to Eichler [6]. I would be remiss if I did not acknowledge my debt accrued by rereading his paper. I have also profited greatly from Weil [8].

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