POSITIVE LINEAR MAPS OF OPERATOR ALGEBRAS

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1. Introduction and basic concepts

1.1. *Introduction.* This paper will be concerned with positive linear maps between C^*-algebras. Motivated by the theory of states and other special maps, two different approaches will be taken. If $\mathfrak{A}$ and $\mathfrak{B}$ are C^*-algebras the set of all positive linear maps of $\mathfrak{A}$ into $\mathfrak{B}$ which carry the identity operator in $\mathfrak{A}$ into a fixed positive operator in $\mathfrak{B}$, is a convex set. The main problem dealt with in this paper will be the study of the extreme points of this convex set. The other approach taken is that of decomposing the maps into the composition of nicely handled ones. A general results of this type is due to Stinespring [20]. Adding a strict positivity condition on the maps he characterized them by being of the form $V^*\varrho V$, where V is a bounded linear map of the underlying Hilbert space into another Hilbert space, and ϱ is a *-representation. Another result of general nature of importance to us is due to Kadison. He showed a Schwarz inequality for positive linear maps between C^*-algebras [11]. Positive linear maps are also studied in [3], [13], [14], and [15].

This paper is divided into eight chapters. In chapter 2 the maps are studied in their most general setting—partially ordered vector spaces. The first section contains the necessary formal definitions and the most general techniques. The last part contains results closely related to what Bonsall calls perfect ideals of partially ordered vector spaces [2]. From chapter 3 on the spaces are C^*-algebras. We first show how close extremal maps are to being multiplicative (Theorem 3.1), and then see that C^*-homomorphisms are extremal (Theorem 3.5), and when the maps generalizing vector states are extremal (Theorem 3.9).

In chapter 4 a geometrical condition stronger than extremality is imposed on the maps. It is shown that for identity preserving maps of an abelian C^*-algebra

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