

THE VALUATION THEORY OF
MEROMORPHIC FUNCTION FIELDS OVER OPEN
RIEMANN SURFACES

BY

NORMAN L. ALLING

M. I. T., Cambridge, Mass., U.S.A.⁽¹⁾

Introduction

With the advent of the generalization of the Weierstrass (product) theorem and the Mittag-Leffler theorem to arbitrary open Riemann surfaces X (due to Florack [6]), the analysis, made by Henriksen [10] for the plane and Kakutani [13] for schlicht domains of the plane, of the maximal ideals in the algebra A of all analytic functions on X can be carried out in general; this will be done in § 1. The residue class field K , associated with free maximal ideals M in A , has been considered by Henriksen [10]. That K has a natural valuation whose residue class field is the complex field C does not seem to have been noticed before. It will be shown in § 1 that the value group of K is a divisible η_1 -group and that every countable pseudo-convergent sequence in K has a pseudo-limit in K : i.e., K is 1-maximal.

Let A_M be the quotient ring of A with respect to M in F , the field of meromorphic functions on X . It will be shown in § 2 that A_M is a valuation ring of F . The value group of A_M will be shown to be a non-divisible near η_1 -group with a smallest non-zero convex subgroup, which is discrete; thus the structure of the prime ideals in A that contain M can be analyzed. It is also shown in § 2 that this valuation on F is 1-maximal.

In § 3 the composite of the place of F , whose valuation ring is A_M and of the place of K , will be shown to be a place of F over C onto C whose valuation is 1-maximal, and whose value group is a non-divisible η_1 -group.

In § 4 the space S of all places of F over C onto C will be considered. Under the weak topology S is compact. Let T be the closure of X in S and let S_A be the places that arise

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