ON THE BOUNDARY THEORY FOR MARKOV CHAINS

BY

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§ 1. Introduction and Summary

The boundary theory of Markov chains, as viewed here, is the study of essential discontinuities (viz., those which are not jumps) of the sample functions. The underlying assumptions are such that these discontinuities form a set of measure zero on the time axis and that for any given time t, the sample function will almost certainly have only jumps within an open interval containing t, reaching the boundary at both ends if at all. Thus it is a question of "how the sample curves manage to go to infinity and to come back from there" (see the preface to [1]). In Paul Lévy's terminology [9], it is a study of "fictitious states". Depending on whether the transition is to or from such a state, it is called a point on the "exit" or "entrance" boundary by Feller ([6], [7]). These ideal boundaries can be formally defined in terms of the R. S. Martin boundary theory (see [4], [5], and [8]), and the question becomes that of a suitable compactification of a discrete set, the denumerable state space of the Markov chain.

In this paper we are mainly concerned with the probabilistico-analytical aspect of the theory rather than the algebraico-topological one, if such a rough distinction may be made. Although the boundary can be defined in the general case and in more than one way, so far only the atomic part consisting of a denumerable number of boundary points has been penetrated in any sense, and substantially so only if their number is finite. It is this part which engages our attention here.

The content of this paper is most directly related to Feller's pioneering work [7]. Indeed, part of the present work arose from an effort to clarify and consolidate his results in probabilistic terms. While Feller regards his problem as one of constructing

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