

VALUE DISTRIBUTION UNDER ANALYTIC MAPPINGS OF ARBITRARY RIEMANN SURFACES

BY

LEO SARIO

University of California, Los Angeles, U.S.A. ⁽¹⁾

I. We consider analytic mappings of an open Riemann surface R into a closed Riemann surface S .

The first and second main theorems of the classical Nevanlinna–Ahlfors [3] theory were generalized in 1960 by S. Chern [4] to an R obtained from a closed surface by omitting a finite number of points. He also referred to a forthcoming paper where, in addition, a finite number of disks are removed.

Chern's elegant work stimulated the present author to look into the question: what can be said about arbitrary open Riemann surfaces R ? In particular, can Nevanlinna's first and second main theorems and any counterpart of Picard's great theorem be established on them? A priori there seemed to be no basis for a conjecture: it was known from classification theory that when the genus becomes infinite, intuition can no longer be relied upon, and surprises are possible. Moreover, although J. Tamura [12] had singled out a class of meromorphic functions with at most two Picard values, M. Heins [5] had exhibited a parabolic Riemann surface with one boundary component, which carried a meromorphic function with an infinite number of Picard values. This rather seemed to speak against any second main theorem in the general case.

In the present paper we propose a new choice for the proximity function and the characteristic function. The first main theorem, the second main theorem, and the defect and ramification relations can then be given for arbitrary Riemann surfaces

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