ON THE CHARACTERISTIC OF FUNCTIONS MEROMORPHIC IN THE UNIT DISK AND OF THEIR INTEGRALS

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1. Introduction

Suppose that $F(z)$ is meromorphic in $|z|<1$ and satisfies $F(0)=0$ there, and that $f(z)=F'(z)$. We define as usual

$$m(r, F)=\frac{1}{2\pi}\int_0^{2\pi}\log^+|F(re^{i\theta})|\,d\theta,$$

$n(r, F)$ as the number of poles in $|z|\leqslant r$ and

$$N(r, F)=\int_0^r \frac{n(t, F)\,dt}{t}.$$

Then

$$T(r, F)=m(r, F)+N(r, F)$$

is called the Nevanlinna characteristic function of $F(z)$. The function $T(r, F)$ is a convex increasing function of $\log r$, so that

$$T(1, F)=\lim_{r\to 1} T(r, F)$$

always exists as a finite or infinite limit. If $T(1, F)$ is finite we say that $F(z)$ has bounded characteristic in $|z|<1$.

Examples show that $F(z)$ may have bounded characteristic in $|z|<1$, even if $f(z)$ does not.(¹) We may take for instance $f(z)$ to be a regular function

$$f(z)=\sum_{n=1}^{\infty}\lambda_n^{\alpha} z^{\lambda_n-1},$$

(¹) The first such example is due to Frostman [3].