Regularity theorems for nondiagonal elliptic systems

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1. Introduction

Recently the close connection between regularity properties of weak solutions of quasilinear elliptic systems of partial differential equations and Liouville-type theorems for such systems has become an object of investigation by several author's ([2], [4], [5], [7], [9]). Up to one exception results concerning regularity properties have been established first and Liouville-type theorems for the corresponding systems have been proved afterwards. Ivert [5] was the first to reverse this relation and to give a negative answer to an open question in regularity theory by using a counterexample to a Liouville-type conjecture due to Meier [8].

Concerning the other way around — to give a *positive* answer to the regularity problem for a certain class of elliptic systems under *weaker* conditions than previously known via Liouville-type theorems — nothing has come to the author's attention. It is the aim of this note to give results of this kind and show that with the same methods it is possible to prove new regularity theorems for weak solutions of quasilinear elliptic systems

$$\frac{\partial}{\partial x_\beta}\left(a_{ij}^{\alpha\beta}(x,u)u_{x_\alpha}^i\right)=f^j(x,u,\nabla u),\quad j=1,\ldots,N,$$

where f may grow quadratically with respect to $|\nabla u|$ and $a_{ij}^{\alpha\beta}(x,u)=A_{ij}^{\alpha\beta}(x)+B_{ij}^{\alpha\beta}(x,u)$ is elliptic as well as $A_{ij}^{\alpha\beta}$, where $A_{ij}^{\alpha\beta}$ is continuous and $B_{ij}^{\alpha\beta}(x,u)$ only measurable, and a relative smallness condition for the measurable part is known. This condition is (in contrast to [3]) not unnecessarily strong but is — restricted to the case $A_{ij}^{\alpha\beta}=\hat{A}^{\alpha\beta}\delta_{ij}$ treated by Sperner [11] — especially independent of the ratio of eigenvalues of $\hat{A}^{\alpha\beta}$ thus improving the results given there. A concrete bound for $B_{ij}^{\alpha\beta}$ to be admissable in the general case is given too.

The results are inspired by recent work of Giaquinta—Modica [3], Kawohl [7] and Meier [9] and are essentially based on some ideas due to Campanato [1].