Stability of Fredholm properties on interpolation scales

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1. Introduction

We begin with a review of the basic notions of interpolation theory. Let $A=(A_0, A_1)$ be an interpolation couple of Banach spaces, i.e. A_0 and A_1 are Banach spaces which are continuously embedded in a Hausdorff topological vector space. The vector spaces $\Delta(A)=A_0\cap A_1$ and $\sum(A)=A_0+A_1$ are also Banach spaces with respect to the norms $\|\cdot\|_{A_0\cap A_1}$ and $\|\cdot\|_{A_0+A_1}$ given by:

$$\|a\|_{A_0\cap A_1} = \max\{\|a\|_{A_0}, \|a\|_{A_1}\}$$

$$\|a\|_{A_0+A_1} = \inf\{\|a_0\|_{A_0}+\|a_1\|_{A_1} \mid a_0\in A_0,\ a_1\in A_1,\ a = a_0+a_1\}.$$

Let $S=\{z\in C \mid 0\leqq \operatorname{Re} z\leqq 1\}$, $S_0=\{z\in C \mid 0<\operatorname{Re} z<1\}$. Given an interpolation couple A, $F(A)$ is defined as the space of all $\sum(A)$-valued analytic functions $f(z)$ on S_0 which are bounded and continuous on S, and which also satisfy for $j=0, 1: f(j+it)$: $R\to A_j$ are continuous and $\lim_{|t|\to\infty} f(j+it)=0$.

We define a norm on $F(A)$:

$$\|f\|_{F(A)} = \max\left\{\sup_t \|f(it)\|_{A_0},\ \sup_t \|f(1+it)\|_{A_1}\right\}.$$

$(F(A); \|\cdot\|_{F(A)})$ is a Banach space.

Definition 1.1. The space A_θ consists of all $a\in\sum(A)$ such that $a=f(\theta)$ for some $f\in F(A)$. Define a norm on A_θ:

$$\|a\|_{A_\theta} = \inf\{\|f\|_{F(A)} \mid f(\theta) = a,\ f\in F(A)\}.$$

Next, one defines another space of analytic functions $H(A)$ as follows. Functions g in $H(A)$ are defined on the strip S with values in $\sum(A)$. Moreover they have the following properties: