On the projective classification of smooth n-folds with n even

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Let $Y \subset \mathbf{P}_{\mathbf{C}}$ be an irreducible, n dimensional, projective variety with a smooth normalization $\alpha: M \to Y$ and let $\mathscr{L} = \alpha^* \mathcal{O}_{\mathbf{P}}(1)_Y$. Recent results of [5], [6], [16] imply that either $(M, \mathscr{L})$ is one of a list of specific, well understood polarized varieties or there is a projective manifold X and an ample line bundle L on X such that:

a) M is the blowup $\pi: M \to X$ of X at a finite set F,
b) $K_M \otimes \mathscr{L}^{n-1} = \pi^*(K_X \otimes L^{n-1})$ where $K_X \otimes L^{n-1}$ is ample and spanned by global sections,
c) $L = [\pi(S)]$ for a smooth $S \in |\mathscr{L}|$, or equivalently $\mathscr{L} = \pi^*(L) \otimes [\pi^{-1}(F)]^{-1}$,
d) $K_X \otimes L^{n-2}$ is semi-ample and big, i.e. some positive power $(K_X \otimes L^{n-2})^t$ is spanned by global sections and the map $\alpha: X \to \mathbf{P}_{\mathbf{C}}$ associated to $\Gamma((K_X \otimes L^{n-2})^t)$ has an n dimensional image.

The pair (X, L) is called the first reduction of $(M, \mathscr{L})$ and is very well behaved, see [12], [14] and [17]. It is easy to convert information between (X, L) and $(M, \mathscr{L})$.

Let $\Phi \circ s = \alpha$ be the Remmert—Stein factorization of the map α (in d) above) where $\Phi: X \to X'$ has connected fibres for a normal projective X', and $s: X' \to \mathbf{P}_{\mathbf{C}}$ is finite to one. There is an ample line bundle $\mathscr{K}$ on X' such that $\Phi^* \mathscr{K} = K_X \otimes L^{n-2}$. The pair $(X', \mathscr{K})$ is known as the 2^{nd} reduction and the map Φ is called the second adjunction map. Such pairs have been studied by the authors [4], [15]. X' has only isolated singularities, is 2-factorial and Gorenstein in even dimensions. Thus for n even, $n \geqq 4$, $\mathscr{K} = K_{X'} \otimes L'^{n-2}$ where for a smooth $A \in |L|$, $2\Phi(A)$ is Cartier, i.e. $[2\Phi(A)]$ is invertible and L' is 2-Cartier. This pleasant circumstance makes the 2^{nd} reduction almost as easy to use as the first reduction when n is even, and allows us to use the results of Fujita [5] in this case. Combining this with a recent result [2] we can push the known classification a good deal further. To state our main result it is useful to recall the notion of the spectral value of a pair.