On isomorphisms between Hardy spaces on complex balls

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1. Introduction

Let B be the unit ball in $\mathbf{C}^n$ and D the unit disc in $\mathbf{C}$. The aim of this paper is to present a new proof that the Hardy spaces $H^1(B)$ and $H^1(D)$ are isomorphic. This became necessary after the discovery that Wojtaszczyk's paper [18] contains a mistake. Part of Wojtaszczyk's proof was based on the intuition that nonisotropic distances on the unit sphere in $\mathbf{C}^n$ and on $\mathbf{R}^{2n-1}$ give "locally similar" metric spaces. This, as we show in Section 5 is not true, hence the argument behind Proposition B of [18] is not valid. Our proof is based on the probabilistic approach developed by Maurey in [10]. We show that if the standard Brownian motion is replaced with the diffusion corresponding to the invariant Laplacian in B, then Maurey's method gives an isomorphic embedding of $H^1(B)$ onto a complemented subspace of a martingale H^1-space which, in turn, is isomorphic to a complemented subspace of $H^1(D)$. Then we can use the first part of Wojtaszczyk's proof, which established that $H^1(D)$ is isometric to a complemented subspace of $H^1(B)$ (see also [1]) and the isomorphism follows by the decomposition principle. The last part of [18], where the atomic H^1-space on $\mathbf{R}^m$ with a nonisotropic distance is studied, is of independent interest and in Section 5 we use it to show that, in spite of the mistaken proof, Proposition B of [18] is true.

The case of $1<p<\infty$ was solved in [1, 18] where it was shown that $H^p(B)$ is isomorphic to $L^p([0, 1])$, hence to $H^p(D)$.

In order to make the paper reasonably self-contained we included some arguments which are fairly strightforward modifications of proofs which can be found in the literature.

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