An analytic algebra without analytic structure in the spectrum

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Introduction

Let A be a uniform algebra. Denote by $\hat{A}$ the algebra of all Gelfand transforms of A and by $M(A)$ the spectrum of A. A is called an analytic algebra provided any function $f \in \hat{A}$ which vanishes on a nonvoid open subset of $M(A)$ is identically zero. In [1] Wermer constructed a new "Stolzenberg example", i.e. he constructed a compact set $Y \subset \mathbf{C}^2$ such that X — the polynomially convex hull of Y — contains no analytic disc and $X \setminus Y \neq \emptyset$. In the present note we want to show that $P(X)$ (X the set in Wermer's example) is an analytic algebra, where $P(X)$ denotes the algebra of all continuous complex-valued functions on X which can be approximated uniformly on X by polynomials.

This seems to be interesting for two reasons. Firstly we get more information about Wermer's example.

Secondly, we learn that the identity theorem is a phenomenon in the theory of uniform algebras which occurs not only in connection with analytic structure. To be more precise we give two definitions.

We say that A has analytic structure in a point $x \in M(A)$ if there is a neighbourhood U of x in $M(A)$, an analytic set V in a polycylinder $P \subset \mathbf{C}^n$ and a homeomorphism $\Phi\colon V \to U$ such that $f \circ \Phi$ is an analytic function on V for all $f \in \hat{A}$.

A satisfies the weak identity theorem in $x \in M(A)$ if there is a (fixed) neighbourhood U of x in $M(A)$ such that $f_{|U} \equiv 0$ for each $f \in \hat{A}$ which vanishes in an arbitrary neighbourhood of x.

It follows from the theory of several complex variables that A satisfies the weak identity theorem in x if A has analytic structure in x. Since we can naturally identify $P(X)$ with $\widehat{P(X)}$ and X with $M(P(X))$, we have an example where the identity theorem does not depend on analytic structure in the spectrum.