The propagation of singularities for pseudo-differential operators with self-tangential characteristics

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0. Introduction

In this paper, we study the propagation of singularities for a class of pseudo-differential operators having characteristics of variable multiplicity. We do not assume the characteristics to be in involution, in the sense that their Hamilton fields satisfy the Frobenius integrability condition. Instead, we assume that the characteristic set is a union of hypersurfaces tangent of exactly order $k_0 \geqq 1$ along an involutive submanifold of codimension $d_0 \geqq 2$. This means that the Hamilton fields are parallel at the intersection, and their Lie brackets vanish of at least order k_0 there. We also assume a version of the generalized Levi condition. One example, with $k_0 = 1$, is the wave operator for uniaxial crystals, i.e. trigonal, tetragonal and hexagonal crystals. The main result is stated in Theorem 1.3, and it shows that the wave front set of the solution is propagated along the union of the Hamilton fields of the characteristic surfaces.

The method of proof is to reduce the operator to a first order diagonalizable system — see Proposition 2.3. By the geometry of the problem and the Levi condition, this can be done using the general symbol classes of the Weyl calculus. For this system the Cauchy problem is well posed, and the parametrix is constructed using Lax' method of oscillatory solutions — see Proposition 3.4. The oscillatory solutions are conormal distributions with non-standard symbols, so we need some calculus lemmas in the appendix. The special symbol classes make it possible to "blow up" the singularity of the characteristics as in [10]. The contributions outside the singularity may then be taken care of, and we are left with solving a micro-local system of pseudo-differential operators along the leaves of the singularity, which is done in Section 4. Finally, the singularities of the parametrix are analyzed in Section 5.