Polynomial approximation in Bers spaces of non-Carathéodory domains

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1. Introduction

The reasonings in our recent work [5] conceals a considerably stronger result than that which we had stated as our main theorem. Specifically, the requirement on the domain D to be a **Carathéodory** domain can be replaced by a weaker assumption, namely that D possesses the so called **Farrell—Markuševič property** (to be defined later). We are grateful to J. Brennan for drawing our attention to this possibility.

We retain the notation of [5]. However, here D stands for an arbitrary bounded simply connected domain, not necessarily a Carathéodory domain. If $a(z)$ is a continuous positive function in D, we denote by $H^p(a:D)$ the class of all analytic functions $f(z)$ in D for which

$$\|f\|_a^p = \iint_D |f(z)|^p a(z)\,dx\,dy < \infty, \quad 0 < p < \infty.$$

Throughout this paper φ will denote a conformal map of D onto the open unit disc U and $\psi = \varphi^{-1}$ will be the inverse mapping. We denote by $\delta_D(z)$ the distance from z to ∂D and $\lambda_D(z)$ stands for the Poincaré metric of D. Here $\lambda_U(w) = (1-|w|^2)^{-1}$ and $\lambda_D(z) = \lambda_U(\varphi(z))\ |\varphi'(z)|$. The fact that $\lambda_D(z)$ is decreasing with D and Koebe's 1/4 theorem imply that

$$1/4 \leqq \lambda_D(z)\delta_D(z) \leqq 1, \quad z \in D. \tag{1.1}$$

We shall confine our attention to those weights $a(z)$ which behave like $\delta_\alpha^D(z)$, $a \in \mathbf{R}$. In view of (1.1) and the conformal invariance of $\lambda_D(z)$, it is more convenient to replace $a(z)$ by $\lambda_D^{-\alpha}(z)$. Specifically, we let

$$t_D = \operatorname{Sup}\{q \in \mathbf{R}\colon \mu_q(D) = \infty\},\ \mu_q(D) = \iint_D \lambda_D^{2-q}(z)\,dx\,dy.$$

Then $1 \leqq t_D \leqq 2$; and, moreover, $t_D = 1$ if ∂D is rectifiable. For these and further properties of t_D see [5].