Some representation theorems for Banach lattices

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0. Introduction

We prove that the notions of a Banach lattice with "quasi-interior elements" and of a cyclic $C(X)$-module are essentially equivalent. This leads to various representation theorems for such Banach lattices. The most interesting result is perhaps that every separable non-atomic Banach lattice, such that the dual space has a weak order unit, can be represented by Lebesgue-measurable functions on the unit interval.

1. The notion of "ideal center" was first introduced in the setting of C^*-algebras by Effros [2] and Dixmier [1]. In [6] W. Wils redefined the notion in the setting of partially ordered spaces. He defines there the ideal center of a partially ordered vector space E to be the set of all endomorphisms of E that are bounded (for the order of operators) by a multiple of the identity operator. We shall denote the center of E by $Z(E)$ (Wils writes Z_E) and we shall use the ideal center in a context which was essentially avoided by Wils. According to Wils "Z_E turns out to be a very useful tool in digging up remnants of lattice structure", and since the main interest of Wils is to study more general partially ordered spaces, he almost entirely avoids the case of Banach lattices. In passing he does however prove that the ideal center of a Banach lattice is always a $C(X)$-space (both as a Banach lattice and as a Banach algebra), and it was observed by Hackenbroch [3] that if the given Banach lattice E has a weak order unit then $Z(E)$ is isomorphic (as a vector lattice) to the order ideal generated by u.

Using this fact Hackenbroch proves strong uniqueness theorems for representation spaces of Banach lattices (having a weak unit). Since thus the "endomorphism algebra" of a Banach lattice with weak order unit contains a "large $C(X)$-space" the lattice itself may be considered as a module over its ideal center.

Let A now be a Banach algebra and let M be a Banach space. We shall then say that M is a left Banach module over A, or a left A-module, if M is a representa-