Generalised parabolic bundles and applications to torsionfree sheaves on nodal curves

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Introduction

Let X be an irreducible nonsingular projective curve over an algebraically closed field. Let E be a vector bundle of rank k and degree d on X. We define generalised parabolic vector bundles (or *GPB's*) by extending the notion of a parabolic structure at a point of X to a parabolic structure over a divisor on X as follows.

Definition 1. A parabolic structure on E over a divisor D consists of 1) a flag $\mathscr{F}$ of vector subspaces of the vector space $E_{|D}=E\otimes O_D$:

$$\mathscr{F}\colon F_0(E) = E_{|D} \supset F_1(E) \supset \ldots \supset F_r(E) = 0$$

2) real numbers $\alpha_1, \ldots, \alpha_r$ (with $0\leqq\alpha_1<\alpha_2<\ldots<\alpha_r<1$) called weights associated to the flag.

Definition 2. A *GPB* is a vector bundle E together with parabolic structures over finitely many divisors D_i.

We define semistability, stability of *GPB's*, study their properties and construct moduli spaces in some important cases. The main results are the following:

Result 1. (Proposition 2.2.) The moduli space P of generalised parabolic line bundles L with $\mathscr{F}$ given by $F_0(L)=L_{x_1}\oplus L_{x_2}\supset F_1(L)\supset o$; $x_1, x_2\in X$, $\dim f_1(L)=1$, is a nonsingular projective variety, it is in fact a $\mathbf{P}^1$-bundle over $\operatorname{Pic} X$.

Result 2. (Theorem 1.) There exists a coarse moduli space $M(k, d, a)$ of equivalence classes of semistable *GPB's* of rank k, degree d and with a parabolic structure over a divisor D of degree 2 given by $\mathscr{F}\colon F_0(E)=E_{|D}\supset F_1(E)\supset o$, $a=\dim F_1E$, weights $(\alpha_1, \alpha_2)=(0, \alpha)$. This space is a normal projective variety of dimension $k^2(g-1)+1+\dim F$, F being the flag variety of flags of type $\mathscr{F}$. If k and d are