A Blaschke-type product and random zero sets for Bergman spaces

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0. Introduction

Let $L^p(\mathbf{D})$ $(p \geqq 1)$ be the Banach space of all measurable functions f on the open unit disk $\mathbf{D}=\{z\in\mathbf{C}\colon |z|<1\}$ such that

$$(0.1) \qquad \|f\|_p = \left\{\int_{\mathbf{D}} |f(z)|^p \, dA(z)\right\}^{1/p} < \infty,$$

where dA is the normalized Lebesgue area measure. Let A^p be the subspace of $L^p(\mathbf{D})$ consisting of analytic functions. The A^p are usually called the Bergman spaces.

Definition. We say that a subset Λ of the disk $\mathbf{D}$ is a zero-set for the space A^p if there exists a nonzero function $f\in A^p$ such that $f|_\Lambda=0$.

The purpose of this paper twofold. First we introduce a Blaschke type product whose factors have an extremal property in A^2 similar to the extremal property enjoyed by the classical Blaschke factors for the Hardy spaces H^p. It will be shown that our Blaschke type products converge for all A^p-zero sets, and they are contractive divisors of zeros for A^2. In the second part of the paper we apply these Blaschke type products (or rather their modification) to obtain a result concerning probabilistic characterization of A^p-zero sets. The probabilistic approach to the study of A^p-zero sets was apparently initiated by Emile LeBlanc [B] who obtained the following result:

Theorem ([B]). *Let $\{r_n\}_{n=1}^{\infty}$ be a sequence in $(0, 1)$ that satisfies the condition:*

$$(0.2) \qquad \limsup_{\varepsilon\to 0} \frac{\sum (1-r_n)^{1+\varepsilon}}{\log 1/\varepsilon} < \frac{1}{2p}.$$

Then for almost all independent choices of $\{\theta_n\}_{n=1}^{\infty}$ the set $\{r_n e^{i\theta_n}\}_{n=1}^{\infty}$ is an A^p-zero set.