Value distributions of entire functions in regions of small growth

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1. Statement of results

Let $f(z)$ be an entire function of finite order ϱ. It is classical (cf. [2, Ch. 4]; [6, Ch. 1] that a *proximate order* $\varrho(r)$ may be associated with $f(z)$ so that the corresponding *indicator* function

$$h(\theta) = \limsup_{r\to\infty} \frac{\log |f(re^{i\theta})|}{r^{\varrho(r)}} \qquad (0 \le \theta \le 2\pi)$$

is continuous, 2π-periodic, and trigonometrically convex. Let $I = (\alpha, \beta)$ be an open interval with

$$h(\theta) \le 0 \quad \alpha \le \theta \le \beta, \tag{1.1}$$

and choose $\theta_0, \alpha < \theta_0 < \beta$. We say that the complex number a is *maximally assumed* near $\{\arg z = \theta_0\}$ if there is some $\varepsilon > 0$ such that for all $\delta > 0$

$$\limsup_{r\to\infty} \frac{n(r, a, \theta_0, \delta)}{r^{\varrho(r)}} \ge \varepsilon; \tag{1.2}$$

here $n(r, a, \theta_0, \delta)$ denotes the number of roots of $f(z) - a$, including multiplicity, in the region $\{|z| < r\} \cap \{|\arg z - \theta_0| < \delta\}$. The set of all maximally assumed values near $\{\arg z = \theta_0\}$ for a given $\varepsilon > 0$ will be denoted by $\mathfrak{Z}(\theta_0, \varepsilon)$.

More generally, for a closed subinterval $I_1 = [\alpha_1, \beta_1]$ of I, let $n(r, a, I_1)$ denote the number of roots of $f(z) - a$, including multiplicity, in the region

$$\{|z| < r\} \cap \{\alpha_1 < \arg z < \beta_1\},$$

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