# Duality of space curves and their tangent surfaces in characteristic $p>0$

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## 0. Introduction

Let $X$ be a nondegenerate complete irreducible curve in projective $N$-space $\mathbf{P}^N$ over an algebraically closed field $k$ of characteristic $p$. Let $\pi\colon \tilde{X}\to X$ be the normalization of $X$ and $\mathfrak{G}$ the linear system on $\tilde{X}$ corresponding to the subspace $V_{\mathfrak{G}}=\mathrm{Image}\,[H^0(\mathbf{P}^N, \mathcal{O}(1))\to H^0(\tilde{X}, \pi^*\mathcal{O}_X(1))]$. Let $\tilde{P}$ be a point on $\tilde{X}$. Since $X$ is nondegenerate, there are $N+1$ integers $\mu_0(\tilde{P})<\ldots<\mu_N(\tilde{P})$ such that there are $D_0, \ldots, D_N\in\mathfrak{G}$ with $v_{\tilde{P}}(D_i)=\mu_i(\tilde{P})$ $(i=0, \ldots, N)$, where $v_{\tilde{P}}(D_i)$ is the multiplicity of $D_i$ at $\tilde{P}$. When $p=0$, the sequence $\mu_0(\tilde{P}), \ldots, \mu_N(\tilde{P})$ coincides with $0, 1, \ldots, N$ except for finitely many points. On the contrary, this is not always valid in positive characteristic. However, F. K. Schmidt [12] (when $\mathfrak{G}$ is the canonical linear system) and other authors [8], [9], [10], [13] (for any linear systems) showed that there are $N+1$ integers $b_0<\ldots<b_N$ such that $\mu_0(\tilde{P}), \ldots, \mu_N(\tilde{P})$ coincides with $b_0, \ldots, b_N$ except for finitely many points.

From now on, we denote by $B(\mathfrak{G})$ the set of integers $\{b_0, \ldots, b_N\}$. Since we take an interest in the invariant $B(\mathfrak{G})$, we always assume that $p>0$.

What geometric phenomena does the invariant $B(\mathfrak{G})$ reflect? Roughly speaking, this invariant reflects the duality of osculating developables of $X$. Let $Y$ be a closed subvariety of $\mathbf{P}^N$. We define the conormal variety $C(Y)$ of $Y$ by the Zariski closure of

$$\{(y, H^*)\in Y\times\check{\mathbf{P}}^N \mid y \text{ is smooth},\ \ T_y(Y)\subset H\},$$

where $\check{\mathbf{P}}^N$ is the dual $N$-space of $\mathbf{P}^N$ and $T_y(Y)$ is the (embedded) tangent space at $y$ to $Y$. The image of the second projection $C(Y)\to\check{\mathbf{P}}^N$ is denoted $Y^*$, which is called the dual variety of $Y$. The original variety $Y$ is said to be reflexive if $C(Y)\to Y^*$ is generically smooth (The Monge—Segre—Wallace criterion; see [6, page 169]). In the previous paper [5], we proved the following theorem.