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Some theorems on absolute neighborhood retracts

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1. In this paper we shall study ANR's (absolute neighborhood retracts). The general problem will be as follows. Suppose we have proved that all ANR's have a certain property. Then we may ask, if this property is characteristic for ANR's, or in other words if it is true that a separable metric space having this property necessarily is an ANR. Thus we shall study necessary conditions for a space to be an ANR, and we shall find that some of these conditions are also sufficient.

Using KURATOWSKI's modification ([7] p. 270) of BORSUK's original definition ([1] p. 222), we mean by an ANR a separable metric space X such that, whenever X is imbedded as a closed subset of another separable metric space Z, it is a retract of some neighborhood in Z.

First, we take up the study of local properties of ANR's. It is known that an ANR is locally contractible (cf. [4] p. 273) and BORSUK proved that local contractability is sufficient for a finite dimensional compact space to be an ANR ([1] p. 240). In a recent paper, however, he has given an example of a locally contractible infinite dimensional space, which is not an ANR [3]. So the question then arises, if the property of a space to be an ANR is a local property. That the answer is affirmative is shown by theorem 3.3. In the case of a compact space this has already been proved by YAJIIMA [10].

Thereafter we prove some theorems on homotopy of mappings into an ANR. Briefly the result can be stated by saying that two mappings of the same space into an ANR which are "near" enough to each other, are homotopic, and that if the homotopy is already given on a closed subset and is "small" enough, then this homotopy is extendable. For a compact ANR we can give an exact meaning to the words near and small in terms of some metric. But the uniformity structure implied by a metric does not seem to be a suitable tool for handling non-compact ANR's. Instead of a metric we therefore use open coverings of the space.

BORSUK proved [2] that any compact ANR X is dominated by a finite polyhedron P. This means that there exists two mappings $\varphi : X \to P$ and $\psi : P \to X$ such that $\psi\varphi : X \to X$ is homotopic to the identity mapping $i : X \to X$. We now prove that the polyhedron P and the mappings φ and ψ can be chosen so that this homotopy between $\psi\varphi$ and i is arbitrarily small, and we show that in this way we get a sufficient condition. This result is generalized in a natural way to non-compact spaces by using infinite locally finite polyhedra. Since these polyhedra are ANR's (see corollary 3.5) we thus see that any ANR is dominated by a locally compact ANR.