ON THE REPRESENTATION OF NUMBERS IN THE FORM $ax^2+by^2+cz^2+dt^2$.[1]

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1. Introduction.

1. 1. The object of the present paper is to treat the problem of the representation of large positive integers in the form $ax^2+by^2+cz^2+dt^2$ (where a, b, c, d are given positive integers) by means of the method introduced into the analytic theory of numbers by G. H. HARDY and J. E. LITTLEWOOD.[2] In my dissertation[3] I have proved an asymptotic formula for the number $r(n)$ of representations of a positive integer n in the form $a_1x_1^2+a_2x_2^2+\cdots+a_sx_s^2$, if $s\geq 5$. The proof of this formula is merely a direct application of the method mentioned above without any new idea. The result is

$$(1.\ 11)\qquad r(n)=\frac{1}{\Gamma\left(\frac{1}{2}s\right)}\frac{\pi^{\frac{1}{2}s}}{\sqrt{a_1a_2\ldots a_s}}n^{\frac{1}{2}s-1}S(n)+O\left(n^{\frac{1}{4}s+\varepsilon}\right)+O\left(n^{\frac{1}{2}s-1-\frac{1}{4}+\varepsilon}\right)$$

for every positive ε. Here $S(n)$ is *the singular series*. Obviously this formula is of no use for the form $ax^2+by^2+cz^2+dt^2$, where $s=4$, so that in this case the approximation of the error term must be improved, if possible. The principal

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[1] An account of the principal results of this paper has been published in the '*Verslagen van de Koninklijke Akademie van Wetenschappen*', Amsterdam, 31 Oct. '25.

[2] For the litterature on this subject I refer to the article of BOHR-CRAMÉR (Die neuere Entwicklung der analytischen Zahlentheorie) in the '*Enzyklopaedie der Mathematischen Wissenschaften*'.

[3] 'Over het splitsen van geheele positieve getallen in een som van kwadraten', Groningen, 1924.