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The complex scaling method for scattering by strictly convex obstacles

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1. Introduction and statement of results

The purpose of this paper is to obtain upper bounds on the number of scattering poles in varying neighbourhoods of the real axis for scattering by strictly convex obstacles with C^∞ boundaries. The new estimates generalize our earlier results on the poles in small conic neighbourhoods of the real axis and include the recent result of Hargé and Lebeau [3] on the pole free region. In fact, one of the new components here is their observation on the choice of the angle of scaling (see Sect. 2).

The starting point of our approach is the same as in [13]: the poles are identified with the square roots of complex eigenvalues of a non-self-adjoint operator obtained by scaling 'all the way to the boundary'. That produces a new elliptic boundary problem for which a semi-classical calculus was developed in [13]. It was then applied to the study of the characteristic values of the scaled operator.

In the present work we adopt a more direct and microlocal approach partly similar to the one used in [9]. By a microlocalization on the boundary we reduce the problem to the study of ordinary differential boundary problem for which a detailed spectral information is available.

We recall that if P is $-\Delta$ on $\mathbf{R}^n\backslash\mathcal{O}$, with the Dirichlet boundary condition, and $\mathcal{O}$ is a bounded subset of $\mathbf{R}^n$ with a connected exterior, then the resolvent

$$(P-\lambda^2)^{-1}: L^2(\mathbf{R}^n\backslash\mathcal{O}) \longrightarrow H^2(\mathbf{R}^n\backslash\mathcal{O})\cap H^1_0(\mathbf{R}^n\backslash\mathcal{O}), \quad \operatorname{Im}\lambda>0,$$

extends to a meromorphic operator

$$(P-\lambda^2)^{-1}: L^2_{\text{comp}}(\mathbf{R}^n\backslash\mathcal{O}) \longrightarrow H^2_{\text{loc}}(\mathbf{R}^n\backslash\mathcal{O})\cap H^1_{0,\text{loc}}(\mathbf{R}^n\backslash\mathcal{O}),$$

for $\lambda\in\mathbf{C}$ or $\lambda\in\Lambda$, the logarithmic plane, when n is odd or even respectively (see [6], [14], [10]). Here $H^k(\mathbf{R}^n\backslash\mathcal{O})$ is the standard Sobolev space and $H^1_0(\mathbf{R}^n\backslash\mathcal{O})$ is the