ON A SPECIAL CLASS OF p-GROUPS

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In the study of p-groups the chief difficulty lies in the fact that the number of such groups is very large. It is therefore of interest to study certain classes of p-groups, and the present paper is devoted to such a topic.

Let G be a group and let x, y be elements of G. We define the commutator $[x, y]$ and the transform x^y by the formulae:

$$[x, y] = x^{-1}y^{-1}xy, \qquad x^y = x[x, y] = y^{-1}xy.$$

For subsets U, V of G, $[U, V]$ denotes the group generated by all commutators $[u, v]$, where $u \in U$, $v \in V$. We define the *lower central series*

$$G \geqslant \gamma_2(G) \geqslant \gamma_3(G) \geqslant \ldots \geqslant \gamma_{i-1}(G) \geqslant \gamma_i(G) \geqslant \ldots$$

of G inductively as follows:

$$\gamma_2(G) = [G, G], \qquad \gamma_i(G) = [\gamma_{i-1}(G), G] \quad (i = 3, 4, \ldots).$$

If there exists an integer k such that $\gamma_k(G) = 1$, then G is said to be *nilpotent*, and if k is the smallest such integer, $k - 1$ is called the *class* of G.

p is to denote a prime number and a p-group is a group of order a power of p. It is well known that all p-groups are nilpotent, and we may therefore speak of the class of a p-group. If m, n are integers and $3 \leqslant m \leqslant n$, it is convenient to denote by CF (m,n,p) the set of all groups G of order p^n and class $m - 1$ in which

$$(\gamma_{i-1}(G) : \gamma_i(G)) = p \quad (i = 3, 4, \ldots m).$$

Similarly ECF (m, n, p) denotes the set of those groups G of CF (m, n, p) in which $G/\gamma_2(G)$ is elementary Abelian. These two classes of groups are to be investigated. Many of our earlier results can also be stated for another class of groups which we denote by NCF (m), and which consists of all nilpotent groups G of class $m - 1$ in which each of the groups $\gamma_{i-1}(G)/\gamma_i(G)$ $(i = 3, 4, \ldots, m)$ is an infinite cyclic group. The general considerations on