ANALYTIC FUNCTIONS AND LOGMODULAR BANACH ALGEBRAS

BY

KENNETH HOFFMAN

Massachusetts Institute of Technology, Cambridge, U.S.A.(1)

1. Introduction

The first part of this paper presents a generalization of a portion of the theory of analytic functions in the unit disc. The theory to be extended consists of some basic theorems related to the Hardy class H^p $(1 \leqslant p \leqslant \infty)$. For example, (i) the theorem of Szegö, Kolmogoroff and Krein on mean-square approximation of 1 by polynomials which vanish at the origin, (ii) the theorems of F. and M. Riesz, on the absolute continuity of "analytic" measures, and on the integrability of $\log |f|$ for f in H^1, (iii) Beurling's theorem on invariant subspaces of H^2, (iv) the factorization of H^p functions into products of "inner" and "outer" functions. The second part of the paper discusses the embedding of analytic discs in the maximal ideal space of a function algebra.

The paper was inspired by the work of Arens and Singer [3; 4], Bochner [6], Helson and Lowdenslager [14; 15], Newman [24], and Wermer [27]. Some of the proofs we employ are minor modifications of arguments due to these authors; however, the paper is self-contained and assumes only standard facts of abstract "real variable" theory, e.g., fundamental theorems on measure and integration, Banach spaces, and Hilbert spaces. In particular, very little knowledge of analytic function theory is essential for reading the paper, since the classical results which are to be generalized are special cases of the theorems here.

The Hardy class H^p $(1 \leqslant p < \infty)$ consists of those analytic functions f in the unit disc for which the integrals

$$\int |f(re^{i\theta})|^p d\theta$$

(1) This research was partially supported by the Office of Scientific Research, United States Air Force.