UNRESTRICTED NILPOTENT PRODUCTS

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Let $A_1, A_2, \ldots, A_i, \ldots$ be a countable sequence of infinite cycles and $\prod_{i=1}^{\infty}{}^{HX} A_i$ denote their unrestricted direct product. Then the following are well known theorems, due in main to Specker [22]:

THEOREM OF SPECKER. *Every countable subgroup of* $\prod_{i=1}^{\infty}{}^{HX} A_i$ *is a free abelian group.*

THEOREM OF SPECKER AND ŁOS. *Let* ψ *be a homomorphism of* $\prod_{i=1}^{\infty}{}^{HX} A_i$ *into a free abelian group. Then there exists a positive integer* m *such that*

$$\psi\Big(\prod_{i=m+1}^{\infty}{}^{HX} A_i\Big) = 1.$$

Our aim is to investigate the corresponding situation in the case of the nilpotent product of infinite cycles. In a similar way one can derive results for the unrestricted soluble product and for the unrestricted third Burnside product both of infinite cycles and of cycles of order three.

Before we can give an outline of our main results, we must first introduce the following

Notation. Let v denote a typical power product of a set of power products of the letters of some fixed alphabet and their formal inverses. These power products are called words. The values of the words obtained by substituting elements from a group G for the above letters of the alphabet, in all possible ways, generate a subgroup of G—the verbal subgroup $V(G)$ of G. The verbal subgroups corresponding to the words

$$[[\cdot\cdot[x_1, x_2], \ldots], x_n],$$